ANSWERS AND COMMENTS

QUESTION 1.1

Since $v = d/t$, the time it takes the Sun to travel a distance d around its orbit at speed v is given by $t = d/v$.

The distance d the Sun travels in one orbit is the circumference of a circle of radius $R = 8.5\,\text{kpc}$, so $d = 2\pi R = 2 \times 3.14 \times 8.5\,\text{kpc} = 53.4\,\text{kpc}$. As the speed is given in km s^{-1} rather than kpc s^{-1}, it is convenient to convert this distance to units of km. Since $1\,\text{pc} = 3.09 \times 10^{13}\,\text{km}$, $d = 53.4 \times 10^3\,\text{pc} \times 3.09 \times 10^{13}\,\text{km pc}^{-1} = 1.65 \times 10^{18}\,\text{km}$.

We can then calculate the time taken to complete one orbit of the Galactic centre as

$$t = d/v = 1.65 \times 10^{18}\,\text{km}/220\,\text{km s}^{-1} = 7.50 \times 10^{15}\,\text{s}$$

The number of seconds in one year is $60 \times 60 \times 24 \times 365.25 \approx 3.16 \times 10^7$. Therefore

$$t = 7.50 \times 10^{15}\,\text{s}/3.16 \times 10^7\,\text{s yr}^{-1} = 2.37 \times 10^8\,\text{yr}$$

Note that since R is given to only two significant figures, we should round the final results to the same accuracy: $t = 7.5 \times 10^{15}\,\text{s}$ (in SI units) and $t = 2.4 \times 10^8\,\text{yr}$ (in years).

QUESTION 1.2

(a) Baade found that the stars in the spheroid of M31 were all red giants, whereas those in the disc of that galaxy were blue stars. Moreover, the red stars in the spheroid of M31 resembled those of Galactic globular clusters, and his observations of the blue stars in the disc of M31 resembled those seen in the plane of the Milky Way. His observations showed that different types of stars occupied different parts of a galaxy.

(b) When a star finishes burning hydrogen in its core, it evolves from the main sequence to become a red giant. In a young population, there are plenty of massive main sequence stars (which are blue), but with time they evolve onto the red giant branch. Hence a population of old stars is dominated by the light of red giants. You can therefore infer that the stars in the spiral arms of M31 and the plane of the Milky Way must be young, whereas the stars in the nuclear bulge of M31 and in Galactic globular clusters must be old.

QUESTION 1.3

The locations to which stars travel within the Galaxy are determined by their motions. For example, stars with higher speeds are more likely to be able to travel to great distances away from the Galactic centre. Hence location is also an indicator of motion.

QUESTION 1.4

As massive stars evolve, they convert hydrogen into helium, helium into carbon and oxygen, and carbon and oxygen into still heavier elements. Some of these freshly synthesized elements are ejected into the interstellar medium when stars reach the end of their lives, so the metallicity of the interstellar medium increases with time. Surviving Pop. II stars are very old and hence formed from material with a low metallicity, whereas Pop. I stars are much younger and hence formed more recently from more metal-rich gas.

QUESTION 1.5

While this view is understandable, it is flawed for the following reason. During their main sequence lifetimes, stars convert H into He, but do not produce additional heavier elements. Not until helium burning begins during the giant phase do they produce carbon and oxygen, and even then it occurs only in the core of the star, and is not observed at the surface until late in the evolution of the star.

QUESTION 1.6

(a) The circumference of a circle of radius R is $d = 2\pi R$, so the distance the Earth travels in each orbit is $d = 2 \times \pi \times 150 \times 10^6 \times 10^3\,\mathrm{m} = 9.425 \times 10^{11}\,\mathrm{m}$. (This should be rounded to three significant figures in the final answer.)

(b) The speed of the Earth, v_E, can be calculated from the known distance, d, around the orbit, and the time taken, $T = 365.25 \times 24 \times 60 \times 60\,\mathrm{s} \approx 3.156 \times 10^7\,\mathrm{s}$

i.e. $v_\mathrm{E} = 9.425 \times 10^{11}\,\mathrm{m}/3.156 \times 10^7\,\mathrm{s} = 2.986 \times 10^4\,\mathrm{m\,s^{-1}}$ (about $30\,\mathrm{km\,s^{-1}}$)

(c) The formula of the rotation curve gives

$$M_\odot = v_\mathrm{E}{}^2 r/G = (2.986 \times 10^4\,\mathrm{m\,s^{-1}})^2 \times 150 \times 10^9\,\mathrm{m}/(6.673 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}})$$
$$= 2.004 \times 10^{30}\,\mathrm{m\,s^{-2}\,N^{-1}\,kg^2}$$

Since $1\,\mathrm{N} = 1\,\mathrm{kg\,m\,s^{-2}}$, we can write

$$M_\odot = 2.004 \times 10^{30}\,\mathrm{m\,s^{-2}\,(kg\,m\,s^{-2})^{-1}\,kg^2} = 2.004 \times 10^{30}\,\mathrm{kg}$$

(d) The distance from the Earth to the Sun was given to three significant figures, whereas all other values are known to higher precision, so the final answer is also known to three significant figures, $2.00 \times 10^{30}\,\mathrm{kg}$.

QUESTION 1.7

(a) To use the equation $M(r) = v^2 r/G$, we need to know the speed the Sun moves, v, the radius of its orbit, r, and G the universal gravitational constant.

$M_\mathrm{MW}(8.5\,\mathrm{kpc}) = v^2 r/G$

$$= (220 \times 10^3\,\mathrm{m\,s^{-1}})^2 \times 8.5 \times 10^3\,\mathrm{pc} \times 3.09 \times 10^{16}\,\mathrm{m\,pc^{-1}}/(6.673 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}})$$
$$= 1.905 \times 10^{41}\,\mathrm{m^2\,s^{-2}\,m\,N^{-1}\,m^{-2}\,kg^2}$$
$$= 1.905 \times 10^{41}\,\mathrm{m^3\,s^{-2}\,(kg\,m\,s^{-2})^{-1}\,m^{-2}\,kg^2}$$
$$= 1.905 \times 10^{41}\,\mathrm{kg}$$

This should be quoted to at most two significant figures, $1.9 \times 10^{41}\,\mathrm{kg}$.

Since $M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$, we can write

$$M_\mathrm{MW}(8.5\,\mathrm{kpc}) = 1.905 \times 10^{41}\,\mathrm{kg}/1.99 \times 10^{30}\,\mathrm{kg}\,M_\odot{}^{-1}$$
$$= 9.6 \times 10^{10} M_\odot$$

(b) As the distance of the Sun from the Galactic centre is given to two significant figures, we could quote the result to this many figures. However, you might wonder whether the result really has that much accuracy because we know the Galaxy does not perfectly meet one of the assumptions of the method – that the mass in the Galaxy is distributed in a way that is spherically symmetric. You might justifiably wonder whether only the first digit is really significant. It would be appropriate to quote the result as $M_\mathrm{MW}(8.5\,\mathrm{kpc}) = 1 \times 10^{11} M_\odot$.

QUESTION 1.8

The rotation curve is a plot of speed against distance from the centre, so the more useful form of the rotation-curve equation is $v(r) = (GM(r)/r)^{1/2}$. To sketch the rotation curve, we need to know how v varies with r.

(a) $M(r) = kr$ so $v(r) = (GM(r)/r)^{1/2}$ becomes $v(r) = (Gkr/r)^{1/2} = (Gk)^{1/2}$ = constant. That is, the speed has the same value, irrespective of the distance from the centre, and hence the rotation curve is a horizontal straight line (it is flat) (see Figure 1.38a).

(b) $M(r) = \rho \times \frac{4}{3}\pi r^3$ so $v(r) = (GM(r)/r)^{1/2}$ becomes $v(r) = (G\rho\frac{4}{3}\pi r^3/r)^{1/2} = (G\rho\frac{4}{3}\pi \times r^2)^{1/2}$ = const $\times\, r$. That is, the speed rises in proportion to the distance from the centre, and hence the rotation curve is a straight line passing through the origin (see Figure 1.38b).

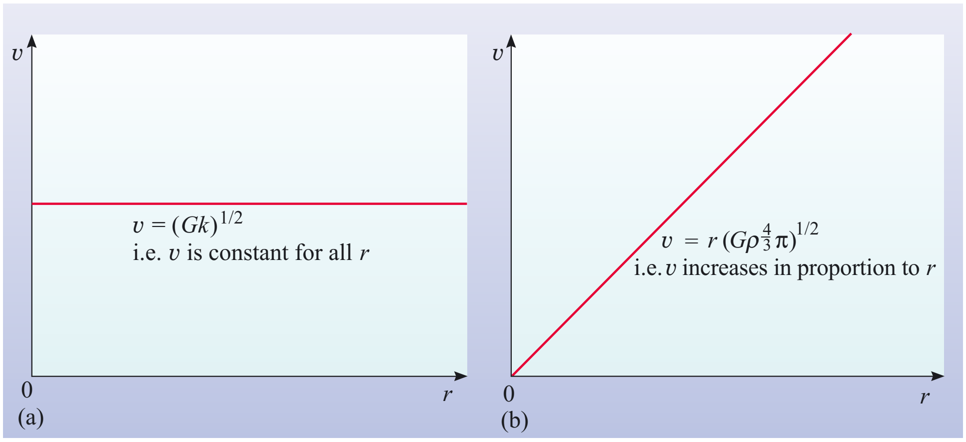

Figure 1.38 Rotation curves for (a) a mass distribution increasing linearly with radius, and (b) a uniform density sphere.

QUESTION 1.9

(a) The time, t, to complete one orbit is given by the distance travelled, d, divided by the speed, v. For a circular orbit, $d = 2\pi r$, where r is the radius of the orbit.

Hence

$$t = d/v = 2\pi r/v = 2\pi \times 4 \times 10^3\,\text{pc} \times 3.09 \times 10^{16}\,\text{m pc}^{-1}/(220 \times 10^3\,\text{m s}^{-1})$$
$$= 3.53 \times 10^{15}\,\text{s}$$

Since the conversion factor from years to seconds is $365.25 \times 24 \times 60 \times 60\,\text{s yr}^{-1} \approx 3.16 \times 10^7\,\text{s yr}^{-1}$, we can write $t = 3.53 \times 10^{15}\,\text{s}/31.6 \times 10^6\,\text{s yr}^{-1} = 1.12 \times 10^8\,\text{yr}$. Therefore, over 4.5×10^9 yr, at 4 kpc the arm would make $4.5 \times 10^9\,\text{yr}/1.12 \times 10^8\,\text{yr}$ = 40 rotations.

(b) Similarly, at 10 kpc an arm would complete one revolution in

$$t = 2\pi \times 1.0 \times 10^4\,\text{pc} \times 3.09 \times 10^{16}\,\text{m pc}^{-1}/(220 \times 10^3\,\text{m s}^{-1}) = 8.83 \times 10^{15}\,\text{s}$$

or $\quad 8.83 \times 10^{15}\,\text{s}/3.16 \times 10^7\,\text{s yr}^{-1} = 2.79 \times 10^8\,\text{yr}$

Over 4.5×10^9 yr, at 10 kpc the arm would make $4.5 \times 10^9\,\text{yr}/2.79 \times 10^8\,\text{yr}$ = 16 rotations.

(c) Over this length of time, the parts at 4 kpc would have made 40 − 16 = 24 more rotations than the 10 kpc part, which means the spiral arm would be wound around the Galaxy 24 times. However, images of other galaxies, (e.g. Figure 1.4b), and even the limited maps of our own Galaxy (e.g. Figure 1.23a), suggest that this does not happen. Spiral arms can usually be traced for *at most* a few circuits of a galaxy.

QUESTION 1.10

If the co-rotation distance is at 15 kpc, then the spiral-arm speed and star speed must be the same at this distance. From Figure 1.13c, the speed here is about 235 km s^{-1}. Since the arms rotate rigidly, the speed must increase linearly with distance from the Galactic centre, so the graph of the rotation curve must resemble Figure 1.39.

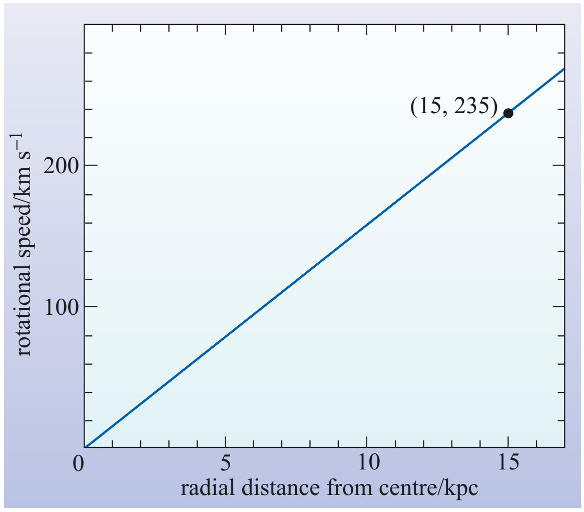

Figure 1.39 Rotation curve for a rigidly rotating spiral pattern that rotates at 235 km s^{-1} at 15 kpc from the Galactic centre. Note that although this rotation 'curve' passes through the centre of the Galaxy, the spiral arms do not extend to radial distances smaller than about 2 kpc from the centre.

The speed of the spiral pattern at 8.5 kpc is therefore $(8.5/15) \times 235$ km s^{-1} = 133 km s^{-1}. Since the Sun is travelling at 220 km s^{-1}, it will approach the wave at 220 km s^{-1} – 133 km s^{-1} = 87 km s^{-1}.

QUESTION 1.11

(a) At the centre of a globular cluster, the number density of stars is 10^4 pc^{-3}, so each star can occupy a volume $1/(10^4$ pc$^{-3})$ which equals 10^{-4} pc^3. To ensure we leave no gaps between adjacent volumes, we must consider cubic spaces rather than spherical ones. Since a cube with sides of length s has a volume s^3, we can write $s^3 = 10^{-4}$ pc^3, so $s = 10^{-4/3}$ pc = 0.046 pc. So the average separation between stars at the centre of a globular cluster is 0.05 pc (to 1 significant figure).

(b) 0.046 pc/1.3 pc ≈ 0.035, so the stellar separation at the centre of a globular clusters is typically 0.035 (i.e. about 1/28) times the distance to alpha Centauri. Clearly stars are packed together much more tightly in globular clusters than they are in the region of the Galaxy near the Sun.

QUESTION 1.12

The given equation can be rearranged to give the logarithm of the distance in terms of the other quantities:

$$\log_{10}(d/\text{pc}) = (m_V - M_V + 5)/5$$

This can be evaluated using the magnitudes provided ($m_V \approx +20.5$, $M_V \approx +0.5$)

$$\log_{10}(d/\text{pc}) = (20.5 - 0.5 + 5)/5 = 5$$

This implies (from the definition of the $\log_{10}$ function) that $d = 10^5$ pc. That is, RR Lyrae stars having $m_V \sim 20.5$ could be seen to a distance of 100 kpc, well out into the most distant parts of the stellar halo.

QUESTION 1.13

Initially, losses to stellar remnants will continue, so the total amount of ISM will decrease below the current 10% of the stellar mass, and for a time, the metallicity will continue to increase due to mass loss from stars. However, as there will be fewer new stars forming in the Galaxy in future because it will have less gas, there will be fewer high-mass stars to enrich the ISM. The ISM will then be replenished only slowly, by the evolution of long lived, low-mass stars. Ultimately the intergalactic medium will be the main means of replenishment, assuming (boldly) that this source is unlimited. Then, the metallicity of the ISM will begin to reflect that of the infalling intergalactic gas.

QUESTION 1.14

High-velocity stars are not part of the disc population (Population I), they really belong to the halo population (Population II). As members of this older population they formed from material which had not yet been enriched in heavy elements by nucleosynthesis and mass loss from stars and supernovae. Therefore they are expected to have, on average, lower metallicity than the Sun.

QUESTION 1.15

According to Figure 1.13c, a star 8.5 kpc from the Galactic centre will have a rotation speed of 220 km s^{-1}. The circumference of a circular orbit of radius 8.5 kpc is $d = 2\pi r = 2\pi \times 8.5 \times 10^3\,\text{pc} \times 3.09 \times 10^{16}\,\text{m pc}^{-1} = 1.65 \times 10^{21}\,\text{m}$.

Thus the time required for a star, such as the Sun, to execute such an orbit is

$$t = \frac{d}{v} = \frac{1.65 \times 10^{21}\ \text{m}}{2.2 \times 10^5\ \text{m s}^{-1}} = 7.50 \times 10^{15}\ \text{s}$$

There are 3.16×10^7 s in one year. So the time required for one complete orbit by the Sun is

$$\frac{7.50 \times 10^{15}\ \text{s}}{3.16 \times 10^7\ \text{s yr}^{-1}} = 2.37 \times 10^8\ \text{yr}$$

Since the Sun has existed for 4.5×10^9 yr, it follows that the number of orbits is

$$\frac{4.5 \times 10^9\ \text{yr}}{2.37 \times 10^8\ \text{yr}} = 19$$

Thus there will have been 19 orbits. (Of course, the two-figure ‘precision’ in this calculation is largely spurious, given the uncertainties that arise in such a calculation.)

QUESTION 1.16

The disc has a radius of about 15 kpc. Thus the area of the disc is $\pi(15\,\text{kpc})^2$ and, since it is $\approx$1 kpc thick, its volume is $\pi(15)^2\,\text{kpc}^3$. By similar reasoning, the optically observable volume of the disc is $\pi(5)^2\,\text{kpc}^2 \times 1\,\text{kpc} = \pi(5)^2\,\text{kpc}^3$. Thus the fraction of the disc’s volume that can be observed is

$$\frac{\pi(5)^2}{\pi(15)^2} = \frac{25\pi}{225\pi} = \frac{1}{9}$$

This limitation is mainly the result of dust in the plane of the Galaxy.

QUESTION 1.17

The Sun is about 4.5×10^9 years old. This is older than all but a very few of the longest-lived open clusters. Thus, even if the Sun was originally part of an open cluster, it would have long since escaped from the cluster. Possible causes of the escape are gravitational disruption (possibly through an encounter with a giant molecular cloud complex), the 'evaporation' of the cluster due to stars occasionally exceeding the escape speed, or simply the dispersive effect of differential rotation over a long period of time.

QUESTION 1.18

In the density wave theory, the spiral pattern moves around rigidly with an unchanging shape, and does not wind up. Matter in the Milky Way revolves differentially, with a longer orbital period for matter at a greater distance from the Galactic centre. Such matter passes into the spiral arms and then out again. Thus the matter highlighting the spiral arms at any time is not permanently present within the arms and thus the arms have no tendency to wind up.

QUESTION 1.19

Tracers of spiral arms include:

- open clusters
- OB associations
- bright HII regions
- dense molecular clouds
- clouds of neutral hydrogen
- T Tauri stars, O and B stars, supergiants and classical Cepheid stars.

QUESTION 1.20

Orange. The brightest stars in a globular cluster will be those at the highest point on the H–R diagram of a cluster of *old* stars. In the globular cluster H–R diagrams the stars in this position are cool red giants, i.e. orange in colour.

QUESTION 1.21

The evidence that the galaxy continues to evolve can be summarized by the following points:

- Star formation is still occurring;
- Enriched gas is returned to the ISM via stellar winds, planetary nebulae and supernovae;
- Infall of intergalactic gas is inferred from gas recycling and high-velocity clouds;
- Some gas from the ISM becomes locked away in the cores of stellar remnants;
- Young stars have higher metallicities than older stars;
- The Sagittarius dwarf galaxy is currently merging with the Milky Way;
- Open star clusters are disrupted by differential rotation of the Galaxy long before most of their stars die;
- Many high-velocity clouds have large velocities towards the Milky Way.

QUESTION 2.1

High-mass main sequence stars, open clusters, HII regions and an abundance of Population I stars (relative to Population II stars), are all indicators of continuing star formation. Since new stars are unlikely to be formed in the absence of cool gas (the raw material needed to make them) it is to be expected that each of these types of object will increase or become more significant in going from ellipticals (which have little cool gas) to spirals, which are actively forming stars in their discs.

QUESTION 2.2

The completed Table 2.1 is shown below.

Property	Ellipticals	Spirals	Irregulars
approximate proportion of all galaxies	$\gtrsim$60%	$\lesssim$30%	$\lesssim$15%
mass of molecular and atomic gas as % of mass of stars	small, 1% say	5–15%	15–25%
stellar populations	Population II	Populations I and II	Populations I and II
approximate mass range	$\sim 10^5 M_\odot$ to $\sim 10^{13} M_\odot$	$\sim 10^9 M_\odot$ to a few times $10^{12} M_\odot$	$\sim 10^7 M_\odot$ to $10^{10} M_\odot$
approximate luminosity range	a few times $10^5 L_\odot$ to $\sim 10^{11} L_\odot$	$\sim 10^9 L_\odot$ to a few times $10^{11} L_\odot$	$\sim 10^7 L_\odot$ to $10^{10} L_\odot$
approximate diameter range [a]	$(0.01–5)\, d_{\mathrm{MW}}$	$(0.02–1.5)\, d_{\mathrm{MW}}$	$(0.05–0.25)\, d_{\mathrm{MW}}$
angular momentum per unit mass	low	high	low

[a] d_{MW}, diameter of Milky Way.

It is important to realize that many of the properties in the table are difficult to determine and that approximate figures are often poorly determined.

QUESTION 2.3

(a) The diameter of the ring $2a$ can be found using Equation 2.2

$$2a = \frac{c\Delta t}{\sqrt{\left(1 - \left(\frac{b}{a}\right)^2\right)}} \qquad (2.2)$$

The time delay is 340 days, so

$c\Delta t = (3.00 \times 10^8\ \mathrm{m\ s^{-1}}) \times (340 \times 24 \times 60 \times 60\ \mathrm{s}) = 8.81 \times 10^{15}\ \mathrm{m}$

The ratio (b/a) can be measured from Figure 2.21 as the ratio between the short and long axes of the ellipse (which is $(2b/2a) = (b/a)$)

(b/a) = (short axis of ellipse)/(long axis of ellipse) = (49 mm)/(69 mm) = 0.710

Substituting these values into Equation 2.2

$$2a = \frac{8.81 \times 10^{15}\ \mathrm{m}}{\sqrt{(1-(0.710)^2)}} = \frac{8.81 \times 10^{15}\ \mathrm{m}}{0.704} = 1.25 \times 10^{16}\ \mathrm{m}$$

So the diameter of the ring around SN 1987A is 1.3×10^{16} m.

(b) To find the distance to a feature of length l that subtends an angle θ as viewed from the Earth, we use the relation $l = d \times (\theta/\text{radians})$ given in Section 2.4.1. Thus

$$d = \frac{l}{(\theta/\text{radians})}$$

From part (a), the diameter of the ring is 1.25×10^{16} m. The angular diameter of the ring is given as 1.66 arcsec, and this needs to be expressed in radians

$$(\theta/\text{radians}) = (\theta/\text{arcsec}) \times (1/57.3) \times (1/3600) = 8.05 \times 10^{-6}$$

It follows that the distance to SN 1987A is

$$d = \frac{1.25 \times 10^{16}\ \text{m}}{8.05 \times 10^{-6}} = 1.55 \times 10^{21}\ \text{m} = \frac{1.55 \times 10^{21}\ \text{m}}{3.09 \times 10^{16}\ \text{m pc}^{-1}} = 5.02 \times 10^{4}\ \text{pc}$$

So the distance to SN 1987A using this method is found to be 50 kpc. (This is consistent with the value of 52 ± 3 kpc that is given in the text.)

QUESTION 2.4

(a) Absorption will reduce the flux received from an object in comparison with the flux that would be measured in the absence of absorption. Thus a distance estimate that is based on applying Equation 2.3 to the flux received when there is absorption will be greater than the value obtained if there were no absorption. So if the effects of absorption are simply ignored, distances will be overestimated.

(b) If the flux is measured over a narrow range of wavelengths, the standard candle method can still be used provided that the luminosity used is that which is emitted over an identical range of wavelengths. Thus, in the visual band (V-band), we could write Equation 2.3 in the form

$$d = [L_V/(4\pi F_V)]^{1/2}$$

Where L_V and F_V are the luminosity and flux in the V-band respectively. (This assumes that radiation does not undergo any significant shift in wavelength between the source and the observer.)

QUESTION 2.5

For a Cepheid with a period of 10 days, Figure 2.25 shows that the average absolute visual magnitude M_V is −4.2.

QUESTION 2.6

The following items of information are needed.

(i) The observed flux density from each supernova at peak brightness. (In practice this would be limited to particular wavebands.)

(ii) A value for the distance, d, to each host galaxy. (In principle this might be based on observations of Cepheid variable periods, but in practice the distances used in these particular cases were based on other bright star observations.)

(iii) An estimate of the amount of radiation absorbed or scattered between the supernova and the flux detector. (Again, in practice this would be limited to a particular waveband.)

The observed flux density should be increased by the amount that was lost due to scattering and absorption, and the resulting total, F, used in conjunction with the distance, d, to find the luminosity, L, where:

$$L = 4\pi d^2 F$$

(If F had been limited to some particular band of wavelengths then L would be limited in the same way. In practice, the calibration of the Type Ia supernova method uses other information besides the three nearby examples mentioned in the question. For example, a number of Type Ia supernovae have been observed in the Virgo cluster of galaxies. Despite the uncertainties about the distance of the Virgo cluster, these observations have also been used in the calibration.)

QUESTION 2.7

The luminosity of a black-body source is given by Equation 2.4

$$L = 4\pi R^2 \sigma T^4 \tag{i}$$

and the flux F is related to luminosity and distance d by Equation 2.3

$$d = \sqrt{\frac{L}{4\pi F}}$$

Squaring both sides gives

$$d^2 = \frac{L}{4\pi F}$$

and rearranging gives

$$F = \frac{L}{4\pi d^2} \tag{ii}$$

So the flux from a black-body source can be found by combining Equations (i) and (ii)

$$F = \frac{4\pi R^2 \sigma T^4}{4\pi d^2} = \frac{R^2 \sigma T^4}{d^2}$$

At two different times, which are denoted by the subscript 0 and 1, respectively, this equation gives

$$F_0 = \frac{R_0^2 \sigma T_0^4}{d^2}$$

$$F_1 = \frac{R_1^2 \sigma T_1^4}{d^2}$$

Thus F_1/F_0 can be found by dividing the right-hand side of second equation by the right-hand side of the first equation. Note that the terms in d and σ cancel out.

$$\frac{F_1}{F_0} = \frac{R_1^2 T_1^4}{R_0^2 T_0^4}$$

As required, this is Equation 2.5.

QUESTION 2.8

Let quantities relating to the two galaxies be denoted by subscripts A and B. The relationship between the velocity dispersions of galaxies A and B can be expressed as

$$(\Delta v)_A = 1.2(\Delta v)_B$$

(a) The luminosities are related to the velocity dispersion according to the Faber–Jackson relation (Equation 2.10)

$$L_A \propto (\Delta v)_A^4$$

$$L_B \propto (\Delta v)_B^4$$

Hence $(L_A/L_B) = ((\Delta v)_A/(\Delta v)_B)^4 = (1.2)^4 = 2.07$

Thus the luminosity of galaxy A is 2.1 times greater than that of galaxy B.

(b) The velocity dispersion, mass M and length scale R are related according to the relation quoted in Section 2.3.2

$$(\Delta v) \propto (M/R)^{1/2}$$

Which can be rearranged to give

$$M \propto (\Delta v)^2 R$$

Thus $M_A \propto (\Delta v)_A^2 R_A$

and $M_B \propto (\Delta v)_B^2 R_B$

These relations imply that

$$(M_A/M_B) = ((\Delta v)_A/(\Delta v)_B)^2(R_A/R_B)$$

But the radii are identical and hence the length scale R is the same for both galaxies, i.e. $R_A = R_B$,

so $(M_A/M_B) = ((\Delta v)_A/(\Delta v)_B)^2 = (1.2)^2 = 1.44$

So the mass of galaxy A is a factor of 1.4 times greater than that of galaxy B.

QUESTION 2.9

The first stage is to rearrange Equation 2.12

$$d = \frac{cz}{H_0}$$

Using the measured redshift of $z = 0.048$, the assumed value of $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, and ensuring that c is expressed in units of $\mathrm{km\,s^{-1}}$, the distance is

$$d = \frac{(3.00 \times 10^5\ \mathrm{km\,s^{-1}}) \times 0.048}{72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}} = 200\ \mathrm{Mpc}$$

So, using Hubble's law, the distance of this galaxy is 200 Mpc.

QUESTION 2.10

To express the Hubble constant in SI units, speed should be expressed in $m\,s^{-1}$ and distance in terms of m.

$$H_0 = 72\,km\,s^{-1}\,Mpc^{-1}$$

$$H_0 = 7.2 \times 10^4\,m\,s^{-1}/(10^6 \times 3.09 \times 10^{16}\,m)$$

$$H_0 = 2.33 \times 10^{-18}\,s^{-1}$$

So a value of H_0 of $72\,km\,s^{-1}\,Mpc^{-1}$ is equivalent to $2.3 \times 10^{-18}\,s^{-1}$.

QUESTION 2.11

(a) The redshift due to a random motion is a Doppler shift. The equation that relates Doppler shift and radial velocity is Equation 1.7

$$\upsilon = c(\lambda_{obs} - \lambda_{em})/\lambda_{em}$$

But $\quad z = (\lambda_{obs} - \lambda_{em})/\lambda_{em}$

So, $\quad \upsilon = cz$

Rearranging

$$z = \upsilon/c$$

So if $\upsilon = 300\,km\,s^{-1}$,

$$z = (300 \times 10^3\,m\,s^{-1})/(3.00 \times 10^8\,m\,s^{-1})$$

i.e. $\quad z = 1.0 \times 10^{-3}$

The typical random velocities of galaxies will give rise to redshifts typically of the order of 0.001. Since the motions are random they are equally likely to be towards us as away from us, and so the redshifts may be negative or positive.

(b) The distance at which the redshifts due to Hubble's law will be a factor of ten greater than the redshifts caused by random motion is found using Hubble's law. From part (a) we know that random motions cause a redshift of 0.001, thus we need to find the distance at which Hubble's law predicts $z = 10 \times 0.001 = 0.01$. Using

$$d = \frac{cz}{H_0}$$

$$d = \frac{(3.00 \times 10^5\ km\,s^{-1}) \times 0.01}{72\,km\,s^{-1}\,Mpc^{-1}} = 42\,Mpc$$

So the redshifts predicted by Hubble's law will be a factor of ten greater than the redshifts due to random motion for distances greater than about 40 Mpc.

QUESTION 2.12

As the stars all grow older together, the massive blue stars would exhaust their core hydrogen first and leave the main sequence. They would then become progressively redder and soon disappear by way of a supernova. (Though this conclusion might be modified in the case of high-mass stars due to the effects of strong stellar winds.) As this process continues, stars of lower and lower mass would gradually leave the main sequence, evolve through the giant stage and eventually end their

lives as white dwarfs. The overall effect would be to reduce the luminosity of the galaxy (because of the growing predominance of lower main sequence stars) and to make it redder. (However, within this general development there are many subtleties that must be considered, mainly arising from the complicated evolution of luminosity and colour of each individual star or type of star.)

QUESTION 2.13

The flattening factor for an elliptical galaxy is $f = (a - b)/a$. For the ellipse shown in Figure 2.3, a is 24 mm and b is 13 mm, so

$$f = \frac{23\,\text{mm} - 13\,\text{mm}}{23\,\text{mm}} = 0.43$$

In assigning a Hubble type to an elliptical galaxy, the number that follows the E is the nearest integer to $10 \times f$. So in this case the appropriate Hubble type would be E4.

QUESTION 2.14

(a) NGC 7479 has wide-flung arms, and there is a bar across its centre; it is an SBc galaxy.

(b) M101 also has wide-flung arms and a relatively small bulge; it is a spiral galaxy of type Sc.

(c) NGC 4449 has no symmetry; it is an irregular galaxy (Irr).

QUESTION 2.15

The ellipsoid is the only three-dimensional shape that presents an elliptical outline to all observers, irrespective of the direction from which it is observed. Oblate and prolate spheroids (and triaxial ellipsoids) are special cases of the general ellipsoid.

QUESTION 2.16

Shortcomings of the standard candle methods include the following.

(i) The difficulty of selecting classes of objects or bodies that have a definite luminosity (i.e. standard candles).

(ii) The difficulty of determining the luminosity of those standard candles (i.e. the calibration problem).

(iii) The likelihood that the standard candles, whatever they may be, will simply be too faint to be seen at all in the more distant galaxies.

(iv) The problems associated with the absorption and/or scattering of radiation along the pathway between the source and the detector. These effects generally reduce the flux density received from the source and make it seem further away than it really is.

(v) The possibility, particularly in the case of spirals, that it may be necessary to take into account the orientation of the galaxy relative to the observer (for example, when using the Tully–Fisher method).

(It is also possible that standard candles observed at great distances (and hence at earlier times, because of the finite speed of light) may not be the same as those relatively nearby objects used for calibration due to evolutionary effects.)

QUESTION 2.17

See Figure 2.44.

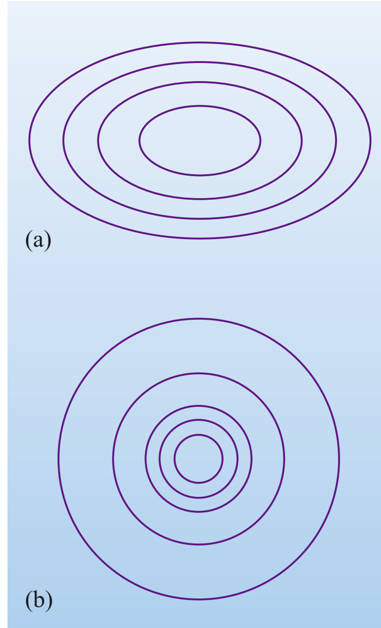

Figure 2.44 The answer to Question 2.17: (a) an E4 galaxy; (b) a face-on S0 galaxy.

QUESTION 2.18

The rotation curve method depends on the stars in a galaxy moving in near circular orbits (Box 1.3). Stars in elliptical galaxies are moving randomly, unlike the fairly orderly rotation of matter in spirals, so there is little net rotation. Consequently the rotation curve analysis cannot be applied to matter in elliptical galaxies.

QUESTION 2.19

(a) M31 is a spiral galaxy, like the Milky Way. Therefore it is to be expected that the central bulge will mainly consist of Population II stars whereas the disc will be dominated by Population I stars. Since these populations are significantly different, it makes sense to model them separately.

(b) In an E2 galaxy, where there is little or no active star formation, the stars will be mainly long-lived types of the sort common in Population II. Thus, lower main sequence stars (i) and red giants (ii) should be well represented, whereas upper main sequence stars (iii) and Cepheid variables (iv) will be rare.

QUESTION 2.20

The correct sequence is (c), (a), (b). In Figure 2.43c, all the stars have formed and the main sequence is well populated, although massive blue stars are much less common than low-mass red stars. A few million years later (Figure 2.43a) the very massive blue stars have burnt themselves out and some of the slightly less massive stars have already left the main sequences and started to become cooler, although no less luminous. Overall, by this stage there has been some reduction in luminosity and a definite change towards a yellower integrated spectrum. After billions of years (Figure 2.43b) even intermediate-mass stars have started to leave the main sequence before entering their giant phase. Overall, owing to the exhaustion of the more massive stars, there has been a further lowering of luminosity and a movement towards a redder spectrum. (Note that the chronological sequence of these H–R diagrams is similar to that shown for the three clusters of stars shown in Figure 1.29.)

QUESTION 3.1

For the Sun's photosphere we have $T = 6000\,\text{K}$ and $m = m_\text{H} = 1.67 \times 10^{-27}\,\text{kg}$.

So the velocity dispersion is given by

$$\Delta v \approx \left(\frac{2kT}{m}\right)^{1/2} = \left(\frac{2 \times 1.38 \times 10^{-23}\,\text{J K}^{-1} \times 6000\,\text{K}}{1.67 \times 10^{-27}\,\text{kg}}\right)^{1/2} = 9960\,\text{m s}^{-1}$$

So hydrogen atoms in the Sun's atmosphere are moving at around $10\,\text{km s}^{-1}$.

Rearranging Equation 3.1 we have

$$\Delta\lambda = \frac{\lambda \Delta v}{c} = \frac{656.3\,\text{nm} \times 9960\,\text{m s}^{-1}}{3.0 \times 10^{8}\,\text{m s}^{-1}} = 0.022\,\text{nm}$$

so the Doppler broadening of the solar Hα line is 0.02 nm (to 1 significant figure). (This is a tiny broadening, about 1 part in 30 000, and rather difficult to observe.)

QUESTION 3.2

The rotation curve shows that the Galaxy is rotating at between roughly 200 and 250 km s^{-1}. Edge-on, this is the approach speed at one extremity and the recession speed at the other. So the line-width that would be observed if the Galaxy were viewed edge-on is 400–500 km s^{-1}.

QUESTION 3.3

The Hβ line has a wavelength of about 485 nm and a width of roughly 6 nm. So the velocity dispersion is

$$\Delta v = \frac{c\Delta\lambda}{\lambda} = \frac{3.0 \times 10^5\ \text{km s}^{-1} \times 6\ \text{nm}}{485\ \text{nm}} \approx 3700\ \text{km s}^{-1}$$

Rearranging Equation 3.2 and putting $m = m_\text{H}$ we have

$$T = \frac{m_\text{H}(\Delta v)^2}{2k} = \frac{1.67 \times 10^{-27}\ \text{kg} \times (3.7 \times 10^6\ \text{m s}^{-1})^2}{2 \times 1.38 \times 10^{-23}\ \text{J K}^{-1}} = 8 \times 10^8\ \text{K}$$

In view of the difficulty of measuring the width of the line, it would be appropriate to give the temperature as approximately 10^9 K. (As is explained in the text following this question, the Hβ emitting region does *not* have such a high temperature.)

QUESTION 3.4

In the optical region (λ = 0.5 μm) galaxy A has $\lambda F_\lambda = 0.5 \times 10^{-29}$ W m^{-2}. For galaxy B, $\lambda F_\lambda = 0.5 \times 10^{-30}$ W m^{-2}. So galaxy A is 10 times brighter in the optical.

In the far-infrared (λ = 100 μm), the upper limit to λF_λ is 10^{-30} W m^{-2} whereas galaxy B has $\lambda F_\lambda = 10^{-28}$ W m^{-2}. The far-infrared flux density of galaxy B is not only greater than that of galaxy A at this wavelength, but also exceeds the flux density at optical wavelengths of both galaxies. On the basis of these (very sparse!) data, it is concluded that galaxy B is the more luminous galaxy.

QUESTION 3.5

The spectrum shows two distinct peaks, one at the red end of the optical (similar to a normal galaxy) and one far into the infrared, near 100 μm. The far-infrared peak is at a similar wavelength to the small peak in a normal spiral galaxy, but it is higher than the optical peak, suggesting that this galaxy emits most of its energy in the far-infrared. There is no significant emission in the UV or X-ray region.

This is not a normal galaxy and you might have guessed that it is an active galaxy. In fact, it is a starburst galaxy. The infrared radiation is coming from dust heated by the continuing star formation and is another distinguishing characteristic of a starburst galaxy, in addition to the strong narrow optical emission lines that you encountered earlier.

QUESTION 3.6

There are several things you may have thought of. Table 3.1 summarizes many of the characteristics and includes some pieces of new information as well. What all active galaxies have in common is a powerful, compact nucleus which appears to be the source of their energy.

Table 3.1 Features of active galaxies compared to those of normal galaxies.

Characteristic		Active galaxies			
	Normal	Seyfert	Quasar	Radio galaxy	Blazar
Narrow emission lines	weak	yes	yes	yes	no
Broad emission lines	no	some cases	yes	some cases	some cases
X-rays	weak	some cases	some cases	some cases	yes
UV excess	no	some cases	yes	some cases	yes
Far-infrared excess	no	yes	yes	yes	no
Strong radio emission	no	no	some cases	yes	some cases
Jets and lobes	no	no	some cases	yes	no
Variability	no	yes	yes	yes	yes

QUESTION 3.7

(a) An angular size limit of 0.1 arcsec corresponds to an angle in radians of

$$(\theta/\text{rad}) = 0.1 \times (1/3600) \times (1/57.3) = 4.8 \times 10^{-7}$$

Multiplying this by the distance shows that the upper limit on the size is $= (50 \times 10^6\,\text{pc}) \times (4.8 \times 10^{-7}\,\text{rad}) = 24\,\text{pc}$.

(b) A week is 7 days which is $7 \times 24 \times 60 \times 60\,\text{s}$. The upper limit from the variability is

$$R \sim \Delta t c = (3.0 \times 10^8\,\text{m s}^{-1}) \times (7 \times 24 \times 60 \times 60\,\text{s}) = 1.8 \times 10^{14}\,\text{m} = 0.006\,\text{pc}$$

(Thus variability constraints provide a much lower value for the upper limit to the size of the AGN than does the optical imaging observation.)

QUESTION 3.8

The relationship between flux density F, luminosity L and distance d is given by Equation 2.3 which can be rearranged to give

$$F = \frac{L}{4\pi d^2}$$

Using this relationship it can be seen that if the AGN is at twice the distance but appears as bright as the normal galaxy in the optical, then it must be emitting four times the optical light of the normal galaxy like our own. If only one-fifth of the AGN's energy is emitted in the optical, then its luminosity is $4 \times 5 = 20$ times that of the normal galaxy like our own, assuming that (as usual) the normal galaxy emits mostly at optical wavelengths. The AGN luminosity is thus about $20 \times 2 \times 10^{10} L_\odot = 4 \times 10^{11} L_\odot$.

QUESTION 3.9

A mass m has a rest energy of mc^2.

(a) If 1 kg of hydrogen were to undergo nuclear fusion to produce helium, the energy liberated would be 0.007 (i.e. 0.7%) of its rest energy:

$$E = 0.007mc^2 = 0.007 \times 1\,\text{kg} \times (3 \times 10^8\,\text{m s}^{-1})^2\,\text{J} = 6 \times 10^{14}\,\text{J}$$

(b) If 1 kg of hydrogen were to fall into a black hole, the energy liberated would be approximately $0.1mc^2 = 0.1 \times 1 \times (3 \times 10^8\,\text{m s}^{-1})^2\,\text{J} = 9 \times 10^{15}\,\text{J}$.

You would expect much *less* energy from the chemical reaction.

QUESTION 3.10

For the Seyfert nucleus, $L = 4 \times 10^{10} L_\odot = 1.6 \times 10^{37}$ W. By Equation 3.5, $Q = L/(0.1c^2)$. Substituting for L,

$$Q = \frac{1.6 \times 10^{37}}{(0.1 \times 9 \times 10^{16})}\,\text{kg s}^{-1} \approx 2 \times 10^{21}\,\text{kg s}^{-1}$$

This can be converted into solar masses per year, by using 1 year ≈ 3×10^7 s, and $M_\odot \approx 2 \times 10^{30}$ kg, giving

$$Q \approx \frac{(2 \times 10^{21} \times 3 \times 10^7)}{2 \times 10^{30}} M_\odot\,\text{yr}^{-1} = 0.03 M_\odot\,\text{yr}^{-1}$$

The Eddington limit places an upper limit on the luminosity for a black hole of given mass.

QUESTION 3.11

Wien's displacement law relates the temperature of a black body to the wavelength at which the spectral flux density has its maximum value. In this case, the dust grains on the inner edge of the torus will be at 2000 K, so their peak emission will be at

$$(\lambda_{\text{max}}/\text{m}) = \frac{2.9 \times 10^{-3}}{(T/\text{K})} = \frac{2.9 \times 10^{-3}}{2000} \approx 1.5 \times 10^{-6}$$

So, λ_{max} is about 1.5 μm.

Grains further from the engine will be cooler, and their emission will peak at longer wavelengths, so the torus can be expected to radiate in the infrared at wavelengths of 1.5 μm or longer. (Note that although the spectrum emitted by dust grains is *not* a black-body spectrum, it is similar enough for the above argument to remain valid.)

QUESTION 3.12

From Equation 3.7 we have

$$r = \left(\frac{L}{16\pi\sigma T^4}\right)^{1/2} = \left(\frac{1 \times 10^{38}\,\text{W}}{16\pi \times (5.67 \times 10^{-8}\,\text{W m}^{-2}\,\text{K}^{-4}) \times (2000\,\text{K})^4}\right)^{1/2}$$

$$\approx 1.48 \times 10^{15}\,\text{m}$$

$$= \frac{1.48 \times 10^{15}\,\text{m}}{3.09 \times 10^{16}\,\text{Mpc}^{-1}} = 4.8 \times 10^{-2}\,\text{pc}$$

Thus, according to this calculation, the radius of the inner edge of the dust torus is 1.5×10^{15} m or 0.05 pc. (A more rigorous calculation, which takes account of the efficiency of graphite grains in absorbing and emitting radiation, gives a radius of 0.2 pc.)

QUESTION 3.13

The NLR is illuminated by radiation from the central engine. As the engine is partly hidden by the dust torus, radiation can only reach the NLR through the openings along the axis of the torus. Any gas near the plane of the torus lies in its shadow and will not be illuminated. The visible NLR would take the form of a double cone of light corresponding to the conical beams of radiation emerging from either side of the torus.

The best view would be from near the plane of the torus, where a wedge-shaped glow would be visible on either side of the dark torus.

QUESTION 3.14

(a) An accreting massive black hole is a hypothesis that has been thought up to account for AGNs. There is really no *conclusive* evidence to support the hypothesis. However, no-one has a *better* idea of how to produce enough power for an AGN in the small volume.

(b) The occurrence of nuclear fusion in the Sun was originally a hypothesis proposed to explain the Sun's energy source. The whole theory of the structure and evolution of stars of different mass and different composition has been based on the nuclear fusion idea. The agreement of this theory with observations is strong confirmation that the nuclear fusion idea is correct.

(c) The laws governing the motion of the planets round the Sun account for all planetary motions ever observed and allow future motions to be predicted. This is the strongest evidence for their correctness. It could even be said that people have conducted experiments by launching spacecraft that are found to move according to these same laws.

QUESTION 3.15

For the gas motion use Equation 3.1, $\Delta\lambda/\lambda = \Delta v/c$, where Δv is the velocity dispersion. Then $\Delta\lambda/\lambda = 2.0\,\text{nm}/654.3\,\text{nm} \approx 0.0030$. Thus the overall spread of internal speeds is $\Delta v \approx 0.0030 \times c \approx 900\,\text{km s}^{-1}$, which is too large for a normal galaxy.

QUESTION 3.16

In the radio wave region, $\lambda = 10^5\,\mu\text{m}$ so

$$\lambda F_\lambda = 10^5\,\mu\text{m} \times 10^{-28}\,\text{W m}^{-2}\,\mu\text{m}^{-1} = 10^{-23}\,\text{W m}^{-2}$$

In the far infrared region $\lambda = 100\,\mu\text{m}$ so that

$$\lambda F_\lambda = 100\,\mu\text{m} \times 10^{-23}\,\text{W m}^{-2}\,\mu\text{m}^{-1} = 10^{-21}\,\text{W m}^{-2}$$

In the X-ray region, $\lambda = 10^{-4}\,\mu\text{m}$ so

$$\lambda F_\lambda = 10^{-4}\,\mu\text{m} \times 10^{-20}\,\text{W m}^{-2}\,\mu\text{m}^{-1} = 10^{-24}\,\text{W m}^{-2}$$

The largest of these λF_λ values is $10^{-21}\,\text{W m}^{-2}$, so we conclude that the far-infrared emission dominates.

QUESTION 3.17

The wavelengths λ are 0.5 μm, 5 μm and 50 μm, therefore the λF_λ values are 5×10^{-28} W m^{-2}, 5×10^{-28} W m^{-2} and 5×10^{-27} W m^{-2}, respectively. The largest of these values is 5×10^{-27} W m^{-2}, so the dominant flux is at 50 μm, which is in the far infrared. The object is likely to be either a starburst galaxy or an active galaxy.

QUESTION 3.18

If the galaxy were active, one would expect to see strong emission lines in the optical and spectral excesses at non-optical wavelengths.

QUESTION 4.1

(a) First, we must convert the angular diameter θ into radians:

$$\theta = 1.9° \times 1/57.3 = 0.0332 \text{ rad}$$

The cluster diameter ($2R$) is given by $d \times \theta$ where d is the distance to the cluster:

$$2R = d \times \theta = 120 \text{ Mpc} \times 0.0332 \text{ radians} = 3.98 \text{ Mpc}$$

So the diameter of the cluster is 4.0 Mpc (to 2 significant figures).

(This is a typical cluster with a radius equal to the Abell radius.)

(b) The cluster is now viewed from distance d = 420 Mpc, and we know that the diameter ($2R$) of the cluster is 3.98 Mpc

$$\theta = 2R/d$$

$$\theta = 3.98 \text{ Mpc}/420 \text{ Mpc} = 9.48 \times 10^{-3} \text{ rad} = 9.48 \times 10^{-3} \times 57.3° = 0.543°$$

So as seen from a distance of 420 Mpc, the angular diameter of the cluster would be 0.54°.

QUESTION 4.2

Using Equation 2.12:

$$z = \frac{H_0}{c} d$$

gives $$d = \frac{cz}{H_0} = \frac{3 \times 10^5 \text{ km s}^{-1} \times 0.25}{72 \text{ km s}^{-1} \text{ Mpc}^{-1}} = 1042 \text{ Mpc}$$

So the distance to is 1.0×10^3 Mpc (to 2 significant figures).

(Note that later in this chapter (Section 4.4.1) you will see that the simple linear relationship between redshift and distance (Equation 2.12) only holds for redshifts less than 0.2. Consequently the result of the answer to this question should be regarded as an *upper limit* to the distance. The deviations from Equation 2.12 for a redshift of 0.25 will be small, so it is reasonable to say that the distance to the furthest cluster in the Abell survey is *approximately* 1000 Mpc.)

QUESTION 4.3

We start by converting the Abell radius of 2 Mpc into metres

$$R_A = 2 \times 10^6 \times (3.09 \times 10^{16})\ \mathrm{m}$$

$$R_A = 6.18 \times 10^{22}\ \mathrm{m}$$

Using Equation 4.1,

$$M = \frac{R_A (\Delta v)^2}{G}$$

$$= \frac{6.18 \times 10^{22}\ \mathrm{m}}{6.67 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}}(5.5 \times 10^5\ \mathrm{m\,s^{-1}})^2 = 2.80 \times 10^{44}\ \mathrm{kg}$$

$$= \frac{2.80 \times 10^{44}\ \mathrm{kg}}{1.99 \times 10^{30}\ \mathrm{kg}} M_\odot = 1.41 \times 10^{14} M_\odot$$

So, using the virial mass method, the mass of the Virgo cluster is found to be $1 \times 10^{14} M_\odot$ (to 1 significant figure).

QUESTION 4.4

Equation 4.2 is an expression for the angular radius of the Einstein ring

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}} \qquad (4.2)$$

This needs to be rearranged to give an expression for the mass M of the cluster. So both sides of Equation 4.2 are squared

$$\theta_E^2 = \frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}$$

and this is then rearranged to give

$$M = \frac{\theta_E^2 c^2 D_L D_S}{4GD_{LS}} \qquad \text{(i)}$$

The question states that we can assume that the cluster is mid-way between Earth and the background galaxies, and so $D_{LS} = D_L$. Substituting this value for D_{LS} into Equation (i) gives

$$M = \frac{\theta_E^2 c^2 D_L D_S}{4GD_L} = \frac{\theta_E^2 c^2 D_S}{4G} \qquad \text{(ii)}$$

D_S is the distance to the background *source* galaxy, which is twice as far away as Abell 2218 itself, so $D_S = 1400$ Mpc.

We must also convert the angle θ_E into radians

$$\theta_E = (1.0/60) \times (1/57.3)\ \mathrm{rad} = 2.91 \times 10^{-4}\ \mathrm{rad}$$

Substituting these values into Equation (ii) gives

$$M = \frac{(2.91\times10^{-4})^2 \times(3\times10^8\ \mathrm{m\,s^{-1}})^2 \times1400\times3.09\times10^{22}\ \mathrm{m}}{4\times6.67\times10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}}$$

$$= 1.24\times10^{45}\ \mathrm{kg}$$

$$M = \frac{1.24\times10^{45}\ \mathrm{kg}}{1.99\times10^{30}\ \mathrm{kg}} M_\odot = 6.23\times10^{14} M_\odot$$

So the mass of the cluster is $6.2\times10^{14}M_\odot$ (to 2 significant figures).

(As mentioned in the text, cluster masses typically lie between 10^{14} and $10^{15}M_\odot$, so this estimate is towards the upper end of the range of cluster masses. This is consistent with the fact that Abell 2218 is one of the richest clusters in the Abell catalogue.)

QUESTION 4.5

The completed table of distances and lengths is given as Table 4.4.

Table 4.4 The scales of different types of cosmic structures. (Completed version of Table 4.3.)

Feature	Distance or length/Mpc
Milky Way (diameter)	0.03
Distance to Large Magellanic Cloud	0.05
Distance to the Andromeda Galaxy	0.8
Extent of the Local Group	~2
Typical diameter of a cluster	~4
Distance to nearest rich cluster (Virgo)	20
Extent of a typical supercluster	30–50
Extent of voids	~60
Scale on which the Universe appears uniform	~200

QUESTION 4.6

Redshift is given by Equation 2.11

$$z = \frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{em}}}{\lambda_{\mathrm{em}}}$$

The original wavelength λ_{em} of the Lyman α line is 121 nm. (Note that although the original wavelength is called λ_{em}, this is of course the wavelength of an *absorption* line due to atomic hydrogen in the intergalactic medium.)

A red-shifted wavelength of $\lambda_{obs} = 372$ nm gives

$$z = \frac{372 - 121}{121} = 2.07$$

From Figure 4.20 a redshift of 2.07 corresponds to a distance of approximately 5.4×10^3 Mpc.

QUESTION 4.7

(a) The Virgo cluster is at a distance of about 20 Mpc. One parsec equals 3.26 light-years, so light will take 3.26 × 15 million years, or about 65 million years to travel 20 Mpc. Therefore we are seeing the Virgo cluster as it was about 65 million years ago. Expressed as a fraction of the age of the Universe this is $(65 \times 10^6 \text{ years})/(1.3 \times 10^{10} \text{ years}) = 5.0 \times 10^{-3}$, or about 0.5% of the age of the Universe.

(b) The mean redshift of galaxies in the SDSS is $z = 0.1$ (Table 4.2). Taking $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$ then the corresponding distance is given by:

$$d = \frac{cz}{H_0} = \frac{3 \times 10^5 \text{ km s}^{-1} \times 0.1}{72 \text{ km s}^{-1} \text{ Mpc}^{-1}} = 416 \text{ Mpc}$$

The time taken for light to travel this distance = $3.26 \times 416 \times 10^6$ years = 1.35 billion years. Thus the mean redshift of the SDSS mapping survey corresponds to light travel times of about 10% of the age of the Universe.

QUESTION 4.8

Since the absorption is due to atomic hydrogen, the structures detected would be biased towards those which contain neutral gas. However we know that the densest regions in the Universe – the cores of rich clusters – are regions where density of neutral gas is very low. This follows from the fact that spiral galaxies are rare in the cores of rich clusters and from the fact that any intracluster gas is at too high a temperature for any hydrogen to remain un-ionized. So studies of the distribution of matter based on Lyman α absoption may be biased towards revealing regions of low density.

QUESTION 4.9

The three methods for determining the total mass of a cluster of galaxies are: (i) using the velocity dispersion of galaxies within the cluster, (ii) from X-ray observations of the intracluster gas, and (iii) from the effect that the cluster has as a gravitational lens.

For the velocity dispersion method it is assumed that the cluster is relaxed or virialized. The determination of mass from X-ray measurements assumes that the intracluster gas is in a state of hydrostatic equilibrium such that the outward pressure of gas balances gravity. The method of using gravitational lensing to determine the cluster mass does not rely on making any assumptions about the physical state of the cluster.

QUESTION 5.1

In a Universe where the baryonic matter is 75% hydrogen and 25% helium (by mass), it has already been shown that there will be 12 hydrogen nuclei for each helium nucleus. Assuming that all the helium is helium-4, it follows that for the two neutrons in each helium-4 nucleus there will be 14 protons (two from the helium-4 nucleus and 12 from the 12 hydrogen nuclei).

Thus, for every 10 neutrons there will be 70 protons. In arriving at this conclusion we have assumed that all hydrogen nuclei consist of a single proton, that all helium nuclei are helium-4, that neutrons and protons have identical mass, and that the universal mix of 75% hydrogen and 25% helium (by mass) is accurate.

QUESTION 5.2

The uniformity of the CMB argues (but does not prove) that the CMB is cosmic. In the case of sunlight, for example, it is clear that there is a nearby source since part of each day is dark when the Earth is between us and the Sun. Most other sources of radiation give similar indications of their local origin by being non-uniform.

QUESTION 5.3

In Section 5.2.3 it was argued that the matter and radiation in the Universe are uniformly distributed at any time. On this basis it is to be expected that the energy and momentum associated with that matter and radiation are also uniformly distributed.

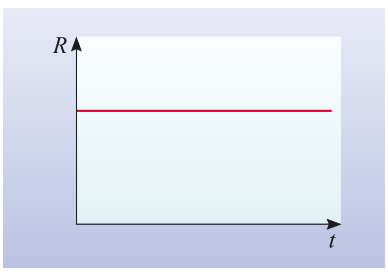

Figure 5.34 R against t graph for the scale factor of the Einstein model.

QUESTION 5.4

In the Einstein model the scale factor R is constant. This means that the graph of R against t will be a horizontal line, as shown in Figure 5.34.

QUESTION 5.5

(a) There are no such ranges, all the FRW models are consistent with the cosmological principle which demands homogeneity and isotropy. (Questions have been raised about the possibility that some models, such as the Eddington–Lemaître model, might evolve into an inhomogeneous state, but these are beyond the scope of this chapter.)

(b) All ranges of k and Λ allow a big bang, but $k = +1$ models with $0 < \Lambda < \Lambda_E$ allow the possibility of universes that began without a big bang. The case $k = +1$, $\Lambda = \Lambda_E$ allows the possibility that the universe might be static (hence no big bang) or that there might not have been a big bang in a non-static universe. Among the limiting cases (like the de Sitter model) that arise as the density approaches zero, there are cases in which the big bang happened an infinitely long time ago.

(c) This is possible for $k = +1$ and $0 < \Lambda \leq \Lambda_E$. (The symbol $\leq$ means 'less than or equal to'.)

(d) There are no ranges that allow the big bang to be associated with a unique point in space. Such an association would violate the cosmological principle. Take good note of this point since it is a widespread misconception to suppose that the big bang was the 'explosion' of a dense primeval 'atom' located at some particular point in space. Rather than thinking of the big bang as an event in space you should think of it as giving rise to space (or rather space–time).

(e) This is true in any model with $k = 0$.

(f) This is true in any model with $k = +1$.

(g) This is true in all models with $k = 0$ or -1. Of course, due to the finite speed of light we can have no direct observational knowledge of those parts of the Universe that are so distant that light emitted from them has not yet reached us.

QUESTION 5.6

The line is described by Hubble's law, $z = (H_0/c)d$, so the gradient of the line represents H_0/c, i.e. the Hubble constant divided by the speed of light in a vacuum. The gradient of the graph is found by dividing the vertical 'rise' Δz of the line by the corresponding horizontal 'run' Δd. In the case of Figure 5.27, this implies

$$\frac{H_0}{c} = \frac{\Delta z}{\Delta d} = \frac{0.15}{0.64 \times 10^3 \text{ Mpc}} = 2.34 \times 10^{-4} \text{ Mpc}^{-1}$$

It follows that

$$H_0 = 3.00 \times 10^5 \text{ km s}^{-1} \times 2.34 \times 10^{-4} \text{ Mpc}^{-1}$$

i.e. $$H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Note that this question is based on artificial data, not real measurements.

QUESTION 5.7

In the Einstein model R does not change with time, so $\dot{R} = 0$ at all times. This implies that for the Einstein model, the Hubble parameter will also be zero at all times. So, $H(t) = 0$. (Einstein produced this model before Hubble's law was discovered, so he did not appreciate the need for an expanding Universe.)

QUESTION 5.8

Figure 5.23 indicates that all the accelerating FRW models correspond to $\Lambda > 0$. So, if q_0 is indeed negative, as recent observations indicate, then we should expect that $\Lambda > 0$ if the real Universe is well described by an FRW model.

QUESTION 5.9

According to the Friedmann equation

$$(\dot{R})^2 = \frac{8\pi G R^2}{3}\left(\rho + \frac{\Lambda c^2}{8\pi G}\right) - kc^2$$

Now $H(t) = \dot{R}(t)/R(t)$, so to get H^2 on the left-hand side of the Friedmann equation, divide both sides by R^2:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\left(\rho + \frac{\Lambda c^2}{8\pi G}\right) - \frac{kc^2}{R^2}$$

i.e. $$H^2 = \frac{8\pi G}{3}\left(\rho + \frac{\Lambda c^2}{8\pi G}\right) - \frac{kc^2}{R^2}$$

as required.

Multiplying both sides of this equation by $3/8\pi G$ gives

$$\frac{3H^2}{8\pi G} = \rho + \frac{\Lambda c^2}{8\pi G} - \frac{3kc^2}{8\pi G R^2}$$

i.e. $$\rho_{\mathrm{crit}} = \rho + \rho_\Lambda - \frac{3H^2}{8\pi G}\frac{kc^2}{H^2 R^2}$$

Dividing both sides by ρ_{crit} gives

$$1 = \frac{\rho}{\rho_{\mathrm{crit}}} + \frac{\rho_\Lambda}{\rho_{\mathrm{crit}}} - \frac{kc^2}{H^2 R^2}$$

i.e. $$1 = \Omega_{\mathrm{m}} + \Omega_\Lambda - \frac{kc^2}{H^2 R^2}$$

Letting $\Omega = \Omega_{\mathrm{m}} + \Omega_\Lambda$ gives

$$\Omega - 1 = \frac{kc^2}{H^2 R^2}$$

If $\Omega_{\mathrm{m}} + \Omega_\Lambda = 1$ then $\Omega = 1$, so $\Omega - 1 = 0$

Therefore $\dfrac{kc^2}{H^2 R^2} = 0$, so $k = 0$.

QUESTION 5.10

Multiplying both sides of the given equation by $\dfrac{-R}{2\dot{R}^3}$ gives

$$\frac{-R\ddot{R}}{\dot{R}^2} = \frac{8\pi G}{3}\frac{R^2}{\dot{R}^2}\left(\frac{\rho}{2} - \rho_\Lambda\right)$$

But $$\frac{R^2}{\dot{R}^2} = \frac{1}{H^2} \quad \text{and} \quad \frac{-R\ddot{R}}{\dot{R}^2} = q$$

So, $$q = \frac{8\pi G}{3H^2}\left(\frac{\rho}{2} - \rho_\Lambda\right)$$

Now, $$\frac{8\pi G}{3H^2} = \frac{1}{\rho_{\mathrm{crit}}}$$

So, $$q = \frac{1}{2}\frac{\rho}{\rho_{\mathrm{crit}}} - \frac{\rho_\Lambda}{\rho_{\mathrm{crit}}}$$

i.e. $$q = \frac{\Omega_{\mathrm{m}}}{2} - \Omega_\Lambda$$

Substituting the currently favoured values $\Omega_{\mathrm{m},0} = 0.3$ and $\Omega_{\Lambda,0} = 0.7$ gives

$$q_0 = 0.15 - 0.7$$

i.e. $$q_0 = -0.55$$

QUESTION 5.11

1916	Einstein's theory of general relativity published. This included the field equations in their original form.
1917	Einstein modified the field equations to include the cosmological constant and published the first (static) cosmological model. The de Sitter model was also published within a year.
1919	Eddington led the eclipse expedition that confirmed the general relativistic prediction that starlight passing close to the edge of the edge of the Sun would be bent.
1922 and 1924	Friedmann published papers on the behaviour of the scale factor in homogeneous and isotropic relativistic cosmologies.
1925	Lemaître introduced the model (later championed by Eddington) that allowed cosmic expansion to follow an indefinitely long period of effectively static behaviour.
1929	Robertson improved and generalized Friedmann's work.
1931	Lemaître formulated his theory of the primeval atom, a crude precursor of the modern big bang theory.
1936	Walker published his independent work on the improvement and generalization of Friedmann's investigations.
1965	Cosmic microwave background radiation discovered.

QUESTION 5.12

The assumptions underpinning the FRW models are that:

- space and time behave in accordance with general relativity; and
- energy and momentum are distributed homogeneously and isotropically on the large scale. (This is the cosmological principle.)

An additional assumption underpinning the Friedmann equation is that:

- the Universe is uniformly filled with a gas of density ρ (that may depend on time).

There are of course other unstated assumptions behind these, such as the belief that space and time are three-dimensional and one-dimensional respectively.

QUESTION 5.13

Since the question concerns FRW models, which are homogeneous and isotropic, it follows that the curvature must be uniform (i.e. the same everywhere and in all directions) at any given time. In a three-dimensional space of (uniform) positive curvature, space will have a finite total volume, straight lines will close back upon themselves, pairs of nearby parallel lines will converge and may meet, any plane triangle will have interior angles that sum to more than 180°, and the circumference of any circle will be less than 2π times its radius.

QUESTION 5.14

In making the step from Equation 5.17 to Equation 5.18 it is stated that

$$\Delta R(t) = \Delta t \times \dot{R}(t)$$

This would be exactly true if the rate of change of R was constant, and it *is* approximately true because we have limited the discussion to cases where Δt is small. However it is not *exactly* true if $\dot{R}$ is changing, i.e. if there is acceleration or deceleration.

QUESTION 5.15

$$H_0 = (72 \pm 8)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

and $$H_0 = (2.3 \pm 0.3) \times 10^{-18}\,\mathrm{s^{-1}}$$

$$\Omega_{\mathrm{m},0} \approx 0.3$$

$$\Omega_{\Lambda,0} \approx 0.7$$

$$q_0 \approx -0.55 \text{ (see answer to Question 5.10)}$$

The age of the Universe t_0 is quoted as slightly less than 14×10^9 years.

To show that the two values of H_0 are equivalent, note that

$$1\,\mathrm{Mpc} = 3.09 \times 10^{22}\,\mathrm{m} = 3.09 \times 10^{19}\,\mathrm{km}$$

Dividing both sides by 1 Mpc shows that

$$1 = 3.09 \times 10^{19}\,\mathrm{km\,Mpc^{-1}}$$

The units make the right-hand side of this equation a complicated way of writing 1. Dividing the first quoted value of H_0 by this conversion factor gives

$$H_0 = \frac{(72 \pm 8)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}}{3.09 \times 10^{19}\,\mathrm{km\,Mpc^{-1}}} = (2.3 \pm 0.3) \times 10^{-18}\,\mathrm{s^{-1}}$$

QUESTION 6.1

The approach to drawing sketches of how the temperature T varies with time t for all Friedmann–Robertson–Walker models with $k = 0$ shown in Figure 5.23, is similar to that adopted in Example 6.1. We shall look at each model in turn.

$k = 0$, $\Lambda < 0$ model

The curve showing $R(t)$ is shown in Figure 6.24a. The behaviour of the scale factor R at times A, B and C, and the inferred behaviour of the temperature T at these times is summarized in Table 6.5. The sketch of $T(t)$ is shown in Figure 6.24b.

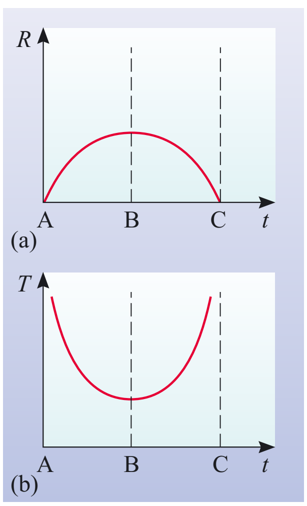

Figure 6.24 (a) Scale factor, and (b) temperature as functions of time for a Friedmann–Robertson–Walker model with $k = 0$, $\Lambda < 0$. A, B and C are times that are referred to in Table 6.5.

Table 6.5 The behaviour of the scale factor R at various times indicated on Figure 6.24a and the inferred behaviour of the temperature T at those times.

Time	Behaviour of R at this time	Behaviour of T at this time
A	$R = 0$	$T = 1/R = \infty$
B	R has increased to a maximum value and now does not vary much with time	T must decrease to some minimum value and also only change slowly with time
C	$R = 0$	$T = 1/R = \infty$

$k = 0$, $\Lambda = 0$ model (the Einstein–de Sitter model)

The curve showing $R(t)$ is shown in Figure 6.25a. The behaviour of the scale factor R at times A, B and C, and the inferred behaviour of the temperature T at these times is summarized in Table 6.6. The sketch of $T(t)$ is shown in Figure 6.25b.

Table 6.6 The behaviour of the scale factor R at various times indicated on Figure 6.25a and the inferred behaviour of the temperature T at those times.

Time	Behaviour of R at this time	Behaviour of T at this time
A	$R = 0$	$T = 1/R = \infty$
B	R is increasing rapidly	T must decrease rapidly
C	R is increasing slowly	T must decrease slowly

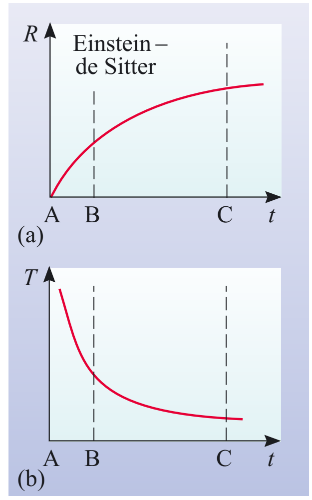

Figure 6.25 (a) Scale factor, and (b) temperature as functions of time for a Friedmann–Robertson–Walker model with $k = 0$, $\Lambda = 0$ (the Einstein–de Sitter model).

$k = 0$, $\Lambda = \Lambda_E$ model

The curve showing $R(t)$ is shown in Figure 6.26a. The behaviour the scale factor R at times A, B, C and D and the inferred behaviour of the temperature T at these times is summarized in Table 6.7. The sketch of $T(t)$ is shown in Figure 6.26b.

Table 6.7 The behaviour of the scale factor R at various times indicated on Figure 6.26a and the inferred behaviour of the temperature T at those times.

Time	Behaviour of R at this time	Behaviour of T at this time
A	$R = 0$	$T = 1/R = \infty$
B	R is increasing rapidly	T must decrease rapidly
C	R is increasing slowly	T must decrease slowly
D	R is increasing to very high values	T must decrease to very small values

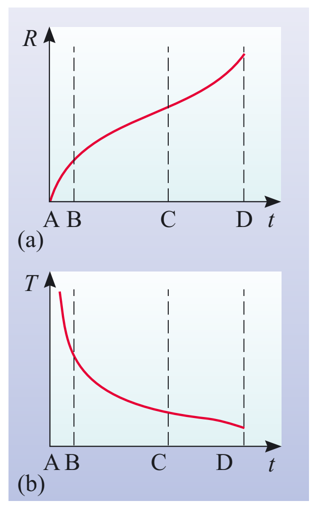

Figure 6.26 (a) Scale factor, and (b) temperature as functions of time for a Friedmann–Robertson–Walker model with $k = 0$, $\Lambda = \Lambda_E$.

QUESTION 6.2

The question states that the current average mass density of luminous and dark matter, $\rho_m \approx 3 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$.

The definition of mass density is that if a volume V contains a mass m, then $\rho_m = m/V$.

The energy density u is defined in an analogous way; if a volume of space V contains an energy E, then $u = E/V$.

To find the energy density due to matter u_m we need to calculate the energy equivalent of the mass m that is contained within the volume V. To do this we use the mass-energy equivalence relation $E = mc^2$,

$$u = E/V = mc^2/V = (m/V)c^2$$

but $(m/V) = \rho_m$, so

$$u = \rho_m c^2$$

Using the value of ρ_m given in the question,

$$u = 3 \times 10^{-27}\,\text{kg m}^{-3} \times (3.00 \times 10^{8}\,\text{m s}^{-1})^2$$

i.e. $$u = 2.7 \times 10^{-10}\,\text{J m}^{-3}$$

Thus the average energy density due to matter is currently $3 \times 10^{-10}\,\text{J m}^{-3}$ (to 1 significant figure).

QUESTION 6.3

The temperature T at time t is given by Equation 6.19

$$(T/\text{K}) \approx 1.5 \times 10^{10}(t/\text{s})^{-1/2}$$

This can be rearranged to give

$$(t/\text{s})^{1/2} \approx 1.5 \times 10^{10}/(T/\text{K})$$

Squaring both sides gives

$$(t/\text{s}) \approx (1.5 \times 10^{10}/(T/\text{K}))^2 \qquad \text{(i)}$$

When $T = 10^6\,\text{K}$,

$$(t/\text{s}) \approx (1.5 \times 10^{10}/(10^6\,\text{K/K}))^2 = (1.5 \times 10^4)^2 = 2.25 \times 10^8$$

$$t = 2.25 \times 10^8\,\text{s} = 2.25 \times 10^8\,\text{s}/(365 \times 24 \times 60 \times 60\,\text{s yr}^{-1}) = 7.13\,\text{yr}$$

So when the temperature was $10^6\,\text{K}$, the age of the Universe was 7 years (to 1 significant figure).

QUESTION 6.4

It is stated in the text that grand unification occurs at an interaction energy of about $10^{15}\,\text{GeV}$. Thus the strong and electroweak interaction became distinct when interaction energies dropped below this value. The corresponding temperature can be found using Equation 6.20, which can be rearranged to give

$$T \sim E/k$$

An interaction energy of $10^{15}\,\text{GeV}$ therefore corresponds to a temperature of

$$T \sim (10^{15} \times 10^9\,\text{eV} \times 1.60 \times 10^{-19}\,\text{J (eV)}^{-1}/(1.38 \times 10^{-23}\,\text{J K}^{-1})$$

$$T \sim 1.16 \times 10^{28}\,\text{K}$$

So, grand unification occurs when the temperature exceeds $10^{28}\,\text{K}$.

In order to calculate the time at which the temperature was $10^{28}\,\text{K}$, we can use Equation (i) from the answer to Question 6.3,

$$(t/\text{s}) \sim (1.5 \times 10^{10}/(10^{28}\,\text{K/K}))^2 = 2.25 \times 10^{-36}$$

So, expressing this to the nearest power of ten, the time at which grand unification occurred was $t \sim 10^{-36}\,\text{s}$.

QUESTION 6.5

(a) (i) Before the decay, the only particle is a single neutron. This has a baryon number of +1. (ii) The baryon numbers of the products of the decay are +1 (proton), 0 (electron) and 0 (electron antineutrino). The baryon number before and after the decay is thus +1 and so baryon number is conserved.

(b) (i) The single neutron has a lepton number of 0. (ii) The lepton numbers of the products are 0 (proton), +1 (electron) and −1 (electron antineutrino). The lepton number before and after the decay is thus 0 and so lepton number is conserved.

(c) (i) A neutron comprises one up and two down quarks (u d d). (ii) A proton comprises two up and one down quark (u u d). The β^--decay reaction thus involves a down quark being transformed into an up quark. Hence β^--decay can be expressed in terms of quarks and leptons as

$$d \rightarrow u + e^- + \overline{\nu}_e$$

QUESTION 6.6

Electron–positron pair production requires an amount of energy given by

$$E = 2m_e c^2$$

(note that the mass of the positron is equal to the mass of the electron m_e). The value of $m_e c^2$ is given in Table 6.3 as being 0.511 MeV, thus

$$E = 2 \times 0.511\,\text{MeV} = 1.02\,\text{MeV} = 1.02 \times 10^6\,\text{eV}$$

So the interaction energy required for electron–positron pair production is 1.02×10^6 eV.

The temperature can be found interaction using Equation 6.20

$$E \sim kT$$

Which can be rearranged as

$$T \sim E/k = (1.02 \times 10^6\,\text{eV} \times 1.60 \times 10^{-19}\,\text{J})/(1.38 \times 10^{-23}\,\text{J K}^{-1}) = 1.18 \times 10^{10}\,\text{K}$$

So the temperature for electron–positron pair production is about 1×10^{10} K. (This is called the *threshold temperature* for electron–positron pairs.)

QUESTION 6.7

The temperature T at time t is given by Equation (i) of the answer to Question 6.3.

(a) When $T = 10^9$ K, then

$$(t/\text{s}) \approx (1.5 \times 10^{10}/(10^9\,\text{K/K}))^2 = 15^2 = 225$$

So the temperature was 10^9 K when $t = 2 \times 10^2$ s (to 1 significant figure).

(b) When $T = 5 \times 10^8$ K, then

$$(t/\text{s}) \approx (1.5 \times 10^{10}/(5 \times 10^8\,\text{K/K}))^2 = 30^2 = 900$$

So the temperature was 5×10^8 K when $t = 9 \times 10^2$ s (to 1 significant figure).

QUESTION 6.8

At around $t \approx 1$ s the ratio n_n/n_p had a value of 0.22. After this time, the dominant reaction affecting free neutrons was β^--decay. If we consider a region of the Universe that contained 100 neutrons at $t \approx 1$ s, then the number of protons in this region would have been $N_p = 100/0.22 = 455$. After this time, the neutrons decayed according to the curve shown in Figure 6.12. The time at which deuterium was first formed was $t = 225$ s. From Figure 6.12 it can be seen that a fraction of 0.78 of the sample of neutrons would remain at this time. Thus the sample would have contained

78 neutrons (N_n = 78). However, due to β^--decay, the number of protons would have *increased* by (100 − 78) = 22, so the total number of protons (N_p) in the sample would have been 455 + 22 = 477. So

$$N_n/N_p = 78/477 = 0.164$$

The ratio N_n/N_p is the same as the ratio of number densities of neutrons and protons n_n/n_p. So at the time that deuterium started to form, $n_n/n_p = 0.16$ (to 2 significant figures).

QUESTION 6.9

The mass fraction in helium (Y) is given by Equation 6.33b. The value of n_n/n_p that was obtained in Question 6.8 is 0.16, so

$$Y = 2\left(\frac{1}{1+(n_p/n_n)}\right) = 2\left(\frac{1}{1+(1/0.16)}\right) = 0.276$$

So the mass fraction in helium is 0.28 (to 2 significant figures).

QUESTION 6.10

The metallicity Z of a sample of material is defined as the mass of the sample that is in metals divided by the total mass of the sample. Since lithium is the only metal (i.e. element with mass number greater than 4) produced in the big bang, the metallicity will be

$$Z = (\text{mass of lithium})/(\text{mass of sample})$$

The mass of the sample can be found from the definition of the hydrogen mass fraction

$$X = (\text{mass of hydrogen})/(\text{mass of sample})$$

$$(\text{mass of sample}) = (\text{mass of hydrogen})/X$$

So the metallicity can be expressed as

$$Z = X \times (\text{mass of lithium})/(\text{mass of hydrogen})$$

This is a useful expression because Figure 6.13 gives the abundance of lithium as the ratio of the mass of lithium to the mass of hydrogen within a sample.

Using the approximation that $X \sim 0.75$,

$$Z = 0.75 \times (\text{mass of lithium})/(\text{mass of hydrogen})$$

For the purposes of this order of magnitude calculation it is reasonable to equate the metallicity to the relative abundance of lithium shown in Figure 6.13. Since the maximum value of the lithium abundance is about 10^{-8}, the maximum metallicity of material formed in the big bang would be of order of magnitude $Z \sim 10^{-8}$.

The oldest observed stars have measured values of $Z \sim 10^{-6}$ (Chapter 1). Thus the metallicity of material formed in the big bang is expected to be at least a factor of 10^2 smaller than the metallicities observed even in the least chemically enriched stars. So the oldest stars cannot be formed from material that has not been subject to some enrichment after the era of primordial nucleosynthesis.

QUESTION 6.11

In all three cases, A, B and C, the value of $\Omega_{b,0}$ can be found by identifying the locations on the curves in Figure 6.13 that correspond to the given abundances. The values of $\Omega_{b,0}$ are shown in Table 6.8.

In cases A and B the lithium abundances correspond to three different values of $\Omega_{b,0}$. In case C, the lithium abundance corresponds to a range of values of $\Omega_{b,0}$. Thus, in all cases, the lithium abundances on their own do *not* allow $\Omega_{b,0}$ to be determined uniquely.

Table 6.8 The values of $\Omega_{b,0}$ determined from the abundances in Question 6.11.

Case	$\Omega_{b,0}$
A	0.006
B	0.07
C	0.02

QUESTION 6.12

The energy required for the ionization of hydrogen is 13.6 eV. Because the number of photons exceeds the number of protons by a factor of 10^9, by analogy with the case of the photodisintegration of deuterium, recombination will only occur once the mean photon energy is a factor of 9.6 lower than the ionization energy (see Section 6.4.1).

So the mean photon energy

$$\begin{aligned}\varepsilon_{\text{mean}} &= (13.6\,\text{eV})/9.6 = 1.42\,\text{eV} = 1.42\,\text{eV} \times 1.60 \times 10^{-19}\,\text{J eV}^{-1}\\ &= 2.27 \times 10^{-19}\,\text{J}\end{aligned}$$

However, $\varepsilon_{\text{mean}}$ is related to the absolute temperature T by Equation 6.27

$$\varepsilon_{\text{mean}} = 2.7kT$$

Thus $$T = \frac{\varepsilon_{\text{mean}}}{2.7k} = \frac{2.27 \times 10^{-19}\,\text{J}}{2.7 \times 1.38 \times 10^{-23}\,\text{J K}^{-1}} = 6092\,\text{K}$$

So recombination occurred at a temperature of 6.1×10^3 K.

(This is, in fact, something of an overestimate. The ionization of hydrogen may occur, not only by an atom in the ground state absorbing a photon with energy greater than 13.6 eV, but also by an atom absorbing a photon such that it is in an excited state, and then absorbing another photon. The effect of this is to lower the temperature at which recombination starts to occur – to about 4500 K.)

QUESTION 6.13

The relationship between temperature and scale factor is given by Equation 6.6,

$$T \propto \frac{1}{R(t)}$$

Thus the relationship between the temperature of the background radiation at the present time T_0 and that at the time of last scattering T_{last} is

$$\frac{T_{\text{last}}}{T_0} = \frac{R(t_0)}{R(t_{\text{last}})}$$

where t_{last} is the time at which the last scattering of photons occurred.

The relationship between redshift and scale factor is given by Equation 5.13. In this case, the time at which the photon is observed is t_0 and the time at which the photon was emitted is t_{last}, so Equation 5.13 can be written as

$$z = \frac{R(t_0)}{R(t_{last})} - 1$$

So $$z = \frac{T_{last}}{T_0} - 1 \qquad \text{(ii)}$$

The question states that $T_{last} = 3.0 \times 10^3$ K, and $T_0 = 2.7$ K, so

$$z = \frac{3.0 \times 10^3\ \text{K}}{2.7\ \text{K}} - 1 = 1110$$

So the redshift at which the last scattering of cosmic background photons occurred is 1.1×10^3 (to 2 significant figures).

QUESTION 6.14

A 'time-line' for the history of the Universe is shown in Figure 6.27.

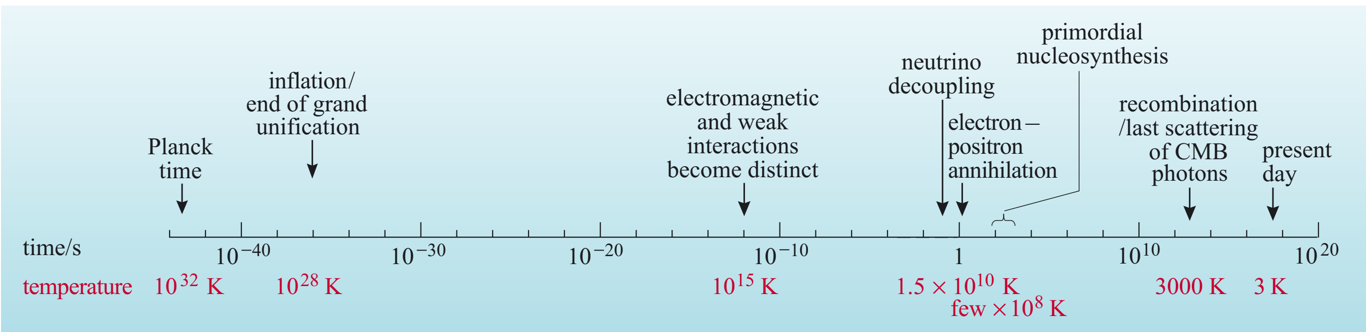

Figure 6.27 The major events in the evolution of the Universe from the Planck time ($t \sim 10^{-43}$ s) to the present day.

QUESTION 6.15

A 'theory of everything' is a theory that accounts for all four of the fundamental interactions of nature in a unified manner. In particular, such a theory would provide a consistent way to describe processes that are currently described by two separate and mutually incompatible theories: the standard model of elementary particles and general relativity (which describes gravitational interactions). Such a theory is needed to describe processes in the very early Universe; before the Planck time (10^{-43} s) the strength of all four fundamental interactions would have been similar, and gravity would have played a role in particle interactions.

QUESTION 6.16

The relationship between scale factor and temperature is given by Equation 6.6, which shows that temperature is inversely proportional to scale factor. Thus a dramatic *increase* in scale factor would cause a dramatic *fall* in temperature. An increase in the scale factor by, for example, a factor of 10^{50}, would cause the temperature to fall by the same factor. Even if the temperature prior to inflation was about 10^{28} K, it would plummet to $10^{28}\ \text{K}/10^{50} = 10^{-32}$ K at the end of inflation. So the temperature at the end of inflation would be expected to be extremely low.

(This might seem at odds with the whole idea of a hot big bang model. How could the Universe that is a tiny fraction of a degree above absolute zero at the end of inflation have proceeded to cause, for example, primordial nucleosynthesis? It turns out that the energy that is released at the end of inflation re-heats the Universe, and it does so as if the cooling due to inflation had never occurred!)

QUESTION 6.17

If the photodisintegration of deuterium required photons of a much higher energy than 2.23 MeV, then, in comparison to the real Universe, a significant amount of deuterium would have been formed at earlier times when the temperature was higher. Thus deuterium would have formed at an earlier time, when the ratio of neutron to proton number densities (n_n/n_p) would have been higher. Most of the neutrons that are present at the time that deuterium production starts, end up in helium nuclei. A higher value of (n_n/n_p) at the start of deuterium production would therefore result in a higher value of the helium mass fraction.

QUESTION 6.18

Following the same reasoning as is used to arrive at Equation (ii) in the answer to Question 6.13, the redshift z is related to the temperature T_{em} of the background radiation at that redshift by

$$z = \frac{T_{em}}{T_0} - 1$$

This can be rearranged to give

$$T_{em} = T_0(z + 1)$$

Inserting the given values leads to

$$T_{em} = (2.73\,\text{K}) \times (2.5 + 1) = 9.56\,\text{K}$$

So, their measurement of the temperature of the cosmic microwave background would be 9.6 K (to 2 significant figures).

(Although we don't have any communication with astronomers anywhere else in the Universe, a similar principle applies to a real observational technique: it is possible to measure the temperature of the cosmic background radiation as experienced by Lyα clouds at redshifts of $z \sim 2$. Such measurements, which are based on detailed analysis of spectral lines, show that the temperature of the cosmic background does increase with redshift in this way.)

QUESTION 7.1

(a) Systematic. This uncertainty would influence the modelling of the lensing galaxy and hence the calculation of the time-of-passage lag, but it would influence repeated measurements in the same way, and would therefore be a systematic effect rather than a random one.

(b) Systematic. Repeated measurements will be affected in the same way each time they are performed.

(c) Random. Performing similar observations several times will lead to different results due to this source of uncertainty.

QUESTION 7.2

Using Hubble's law $d = cz/H_0$, with $z = 1$ and $H_0 = 72\,\text{km s}^{-1}\,\text{Mpc}^{-1}$ (the HST value), gives

$$d = (3.0 \times 10^5\,\text{km s}^{-1})/(72\,\text{km s}^{-1}\,\text{Mpc}^{-1})$$

i.e. $$d = 4.2 \times 10^3\,\text{Mpc}$$

The simple form of Hubble's law used above should not be trusted as a reliable indicator of distances at such large redshift, since it ignores the effect of any acceleration or deceleration in the rate of cosmic expansion. For instance, from the more accurate Equation 7.4, it can be seen that if $q_0 = 0$, then d would increase to 150% of the quoted value. In either case, this would be very much larger than the diameter of the local supercluster, which was described in Chapter 4 as being 20 to 50 Mpc across.

QUESTION 7.3

In the supernova cosmology measurements the emphasis was on determining the magnitude of distant Type Ia supernovae relative to nearer ones. For this purpose it is only necessary to measure the apparent magnitudes of the supernovae rather than the absolute magnitudes. To measure the Hubble constant we would also need to know the absolute magnitudes, in order to determine the distances.

QUESTION 7.4

From Figure 7.11, the lowest point on the 99% confidence contour corresponds to $\Omega_{\Lambda,0} = 0.1$, while the highest point on that contour corresponds to $\Omega_{\Lambda,0} = 2.5$. In contrast, the points at which the 99% contour crosses the 'flat geometry' line correspond to $\Omega_{\Lambda,0} = 0.6$ and $\Omega_{\Lambda,0} = 0.9$.

QUESTION 7.5

The claim that 'baryons only account for somewhere between about a fifteenth and a sixth of the total density of matter in the Universe' was based on Equation 7.13, according to which

$$0.02 \leq \Omega_{\text{b},0} \leq 0.05$$

The widely favoured value for $\Omega_{\text{m},0}$, based on supernova cosmology and measurements of the cosmic microwave background radiation is

$$\Omega_{\text{m},0} \approx 0.3$$

This suggests that

$$0.067 \leq \frac{\Omega_{\text{b},0}}{\Omega_{\text{m},0}} \leq 0.167$$

i.e. between a fifteenth and a sixth.

QUESTION 7.6

The dipole anisotropy is a large-scale non-uniformity in the CMB, as observed from Earth. It causes the direction of maximum intensity (or effective temperature) to be diametrically opposite the direction of minimum intensity (or temperature), with the intensity increasing progressively from the minimum to the maximum. The dipole anisotropy is caused by the motion of the Earth relative to the frame of reference in which the CMB shows large-scale isotropy. (In other words the dipole anisotropy is a result of the Doppler effect.)

The existence of the dipole anisotropy is implicitly acknowledged in Point 1 by saying that the radiation (CMB) is 'intrinsically' uniform to better than one part in 10 000. It is more explicitly recognized in the next sentence by the inclusion of the phrase 'after correcting for effects due to the motion of the detector'.

In Point 2, the word 'intrinsic' is again used to indicate 'after correcting for effects due to the motion of the detector'.

Point 5 also includes the word 'intrinsic' with the same implication.

QUESTION 7.7

The predictions are based on the general principles of big bang cosmology and specific assumptions about the values of the cosmological parameters. Since the predictions don't relate to specific features such as the exact region of sky being observed, or the detailed history of that region, the predictions cannot be expected to show anything more than 'general' agreement even when the appropriate values for the cosmological parameters have been chosen.

QUESTION 7.8

From Figure 7.24, the second and third peaks are located at $l \approx 500$ and $l \approx 800$, respectively.

From Equation 7.16, these values correspond to angular scales $\theta = 0.36°$ and $\theta = 0.23°$, respectively.

The maximum angular power associated with these peaks is roughly $2500\,(\mu K)^2$ in each case.

QUESTION 7.9

(a) The CMB anisotropy measurements indicate that $\Omega_{\Lambda,0} + \Omega_{m,0} = 1.02 \pm 0.02$.

However, Type Ia supernovae, mass-to-light ratios and CMB measurements all indicate that $\Omega_{m,0}$ is much less than 1, probably about 0.3. This implies that $\Omega_{\Lambda,0} \approx 0.7$. Thus the energy associated with the cosmological constant (dark energy) dominates the density of the Universe. The nature of this energy is not well understood at the time of writing.

(b) Among other results discussed in earlier chapters, the growth of mass-to-light ratios with the scale of observations indicates that most of the matter in the Universe is dark matter.

(c) Although observations indicate that $\Omega_{m,0} \approx 0.3$, estimates of $\Omega_{b,0}$ based on primordial nucleosynthesis constraints, baryon inventories and CMB measurements, indicate that $\Omega_{b,0} \approx 0.04$. Hence most of the (dark) matter must be non-baryonic.

QUESTION 7.10

Even from the incomplete account given in this chapter it is clear that space technology has profoundly influenced observational cosmology. This is clear from:

- the role of the HST in facilitating the determination of the Hubble constant from Cepheid variable observations;
- the use of the HST to study some of the faintest (and therefore most distant) Type Ia supernova candidates used in the determination of the current value of the deceleration parameter;
- the role of space-based X-ray observations in the determination of cluster masses that are an essential ingredient in measurements of the mass-to-light ratio on large scales;
- the additional role of X-ray and other space-based observations in compiling a baryon inventory;
- the importance of COBE in confirming the black-body nature of the CMB spectrum, determining the mean temperature of the radiation and revealing the existence of anisotropies in that temperature; and
- the role of WMAP and the expected role of the Planck mission in measuring the CMB angular power spectrum with great precision so that the key cosmological parameters can be precisely determined.

QUESTION 7.11

Terrestrial observations, including those performed from planes and balloons, are still of enormous importance in cosmology, despite the many advances that can be attributed to space technology. Examples of important ground-based or air-based experiments include:

- observations of gravitational lensing using optical and radio telescopes (although important results have also come from the HST);
- studies of supernovae carried out by large telescopes at sites such as Mount Palomar and, more recently, Cerro Tololo and Hawaii (Keck);
- observations of various kinds used in establishing mass-to-light ratios, baryon inventories and the relative abundances of light elements that are needed to check primordial nucleosynthesis predictions;
- a range of CMB observations including the initial breakthrough by Penzias and Wilson, the discovery of the dipole anisotropy and the anisotropy measurements carried out by BOOMERanG and various other projects; and
- the large-scale galaxy surveys that are helping to refine our knowledge of the galaxy power spectrum.

QUESTION 8.1

Some examples of known objects that could make up the dark matter are:

Brown dwarfs – stellar bodies whose masses are too low for hydrogen burning to have commenced in their cores.

Stellar remnants – cooled white dwarfs, neutron stars, black holes

Minor bodies – planets, asteroids, comets

Note that all of these candidates contain baryonic matter. Strictly speaking, black holes should not be counted as being a form of baryonic matter, but we shall treat them as such on the basis that they were probably formed from baryonic matter.

QUESTION 8.2

The answer is given in Table 8.2.

Table 8.2 The completed Table 8.1 (Question 8.2).

Dark matter candidate	Baryonic	Non-baryonic	MACHO	WIMP	Cold[a]	Hot[a]
brown dwarfs	✓		✓			
neutrinos		✓		✓		✓
neutron stars	✓		✓			
black holes	✓		✓			
neutralinos		✓		✓	✓	

[a] Note that the terms hot and cold only apply to WIMPs, since they refer to the speed of particles at the time of decoupling and not to temperature in the conventional sense.

QUESTION 8.3

(a) An antihydrogen atom (made from an antiproton and a positron) would have the same energy levels as normal hydrogen and would absorb and emit photons in the same way. The photons themselves would be identical to photons produced in our own Galaxy. The same applies to all other atoms. So the spectrum of an antigalaxy would look much the same as an ordinary galaxy.

(b) If the galaxy really is an antigalaxy, then somewhere between the galaxy and our own Milky Way there must be a boundary between matter and antimatter. We would expect annihilations to be taking place at the boundary, perhaps in the intergalactic medium, and the boundary could revealed by a search for γ-rays. In fact, the absence of such γ-radiation is strong evidence that there are no significant amounts of antimatter in our part of the Universe.

QUESTION 8.4

You didn't really expect to find an answer to this one, did you?

QUESTION 8.5

The end-of-chapter summary provides an overview of the current status of the cosmological problems that were outlined at the beginning of the chapter.